	#	Mark
MAT344 Term Test # 1 **February 13, 2018**	**1** [4 points]	
Last name	**2** [3 points]	
First name	**3** [4 points]	
ID	**4** [5 points]	
Section	**5** [4 points]	
	Total	

1. Let C be the arc of the circle $|z| = 5$ that lies in the 3rd quadrant. Show that

$$\left| \int_C \frac{dz}{z^2 + 16} \right| \leq \frac{5\pi}{18}$$

2. Find the images of the domain $\{z : \text{Re}(z) > 0 , \text{Im}(z) > 0\}$ for both of the branches of $\sqrt{z}$ and solve $z^{-\frac{1}{2}} = i$.

3. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{2^n}{n^2} z^n$

4. Show that $u(x, y) = x^3 - 3xy^2$ is harmonic on $\mathbb{C}$ and find a harmonic conjugate $v(x, y)$ of $u(x, y)$ such that $v(0, 0) = 2$.

5. Calculate fully justifying

$$a) \int_\Gamma \frac{\sin(z)}{z^2} dz , \quad b) \int_\Gamma \frac{dz}{z - 1} ,$$

where Γ is the curve on the picture below. Solve without Cauchy Theorems.
  Hint for part b): add to Γ a small circle C around 1 oriented clockwise, split the resulting curve into several simple curves and use Green's Theorem for the real and the imaginary parts of the integrand. Complete the calculation by explicitly evaluating the integrand on the same circle C oriented counterclockwise.